Trade Dynamics

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Very Preliminary: (Permanently) Work in Progress
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Overview:

Trade and Growth: Learning-By-Doing (LBD)

Trade and Growth: Factor Accumulation

Trade and Growth: Innovation

Growth and International Technology Diffusion

Short-Run versus Long-Run: Models with Structural Adjustments

Throughout Part V, it is assumed there is no international lending/borrowing, so that the net trade balance must be equal to zero, every period. Intertemporal trade will be discussed in Part VI.

Dynamic External Economies of Scale: Learning-By-Doing (LBD) or Knowledge Spillovers

- In Part IV, we studied external economies as a way of introducing increasing returns without departing from the competitive framework. However, one justification for such external economies, learning and knowledge spillovers would naturally call for dynamic extensions.

- We now study trade models with such dynamic external economies, where the productivity of a sector, a country, etc, improves as producers learn from the experiences by other producers in the past.

- As in the static external economies, these externalities may be classified according to the scope of externalities.
  - *Geographical Spillovers*: local (country- or region-specific) or global in scope.
  - *Sectoral Spillovers,* whether learning is specific to the industry where it has been acquired or spillovers to other industries.

- In addition, there is another important distinction, whether the potential for productivity gains through LBD is *unbounded* or *bounded*.

Unbounded, Country-Specific, Sector-Specific LBD

Two Countries: Home (L) and Foreign (L^*)

Two (Tradeable) Intermediate Input Sectors: Labor is the sole input, and its productivity is given by:

$A_j(t)/a_j$ (j = 1, 2) $A_j^*(t)/a_j^*$ (j = 1, 2)

a_j , a_j^*: time-invariant, reflecting the inherent advantages,
$A_j(t)$, $A_j^*(t)$: time-variant, reflecting the learning components.

The (Nontradeable) Final Good Sector: Cobb-Douglas Technologies:

$$Y = \left(\frac{C_1}{\alpha}\right)^{\alpha}\left(\frac{C_2}{1-\alpha}\right)^{1-\alpha}; \qquad Y^* = \left(\frac{C_1^*}{\alpha}\right)^{\alpha}\left(\frac{C_2^*}{1-\alpha}\right)^{1-\alpha}$$

where C_j (C_j^*): input-j (j = 1, 2) used in the Home (Foreign) final good production. The cost functions are $(p_1)^{\alpha}(p_2)^{1-\alpha}$ and $(p_1^*)^{\alpha}(p_2^*)^{1-\alpha}$ where p_j (p_j^*) is the price of input-j.

Learning-By-Doing (Knowledge) Spillovers:
One may write that labor productivity in each sector changes with the experiences accumulated in different sectors and different countries, as follows:

$$A_j = A_j(Q_1, Q_2, Q_1^*, Q_2^*); \qquad A_j^* = A_j^*(Q_1, Q_2, Q_1^*, Q_2^*)$$

where $Q_j (Q_j^*)$ is the experience in Home (Foreign) sector-j.

Here, let $Q_j(t) = \int_0^t X_j(s)ds + Q_j(0)$ and $Q_j^*(t) = \int_0^t X_j^*(s)ds + Q_j^*(0)$ and

$$A_j(t) = \delta_j Q_j(t), \qquad A_j^*(t) = \delta_j Q_j^*(t) \qquad\qquad (j = 1, 2)$$

where $\delta_j \geq 0$ is the learning speed in sector-j. By letting "•" denote the time derivative,

$$\dot{A}_j(t) = \delta_j X_j(t) \quad \text{and} \quad \dot{A}_j^*(t) = \delta_j X_j^*(t). \qquad\qquad (j = 1, 2)$$

Each sector improves its productivity (or accumulates its knowledge capital) at the rate proportional to its own production.

Notes:

- No sector learns from the production in the other sector. Learning is (completely) *sector-specific.*

- No country learns from the production of the other country. Learning is (completely) *country-specific.*

- Productivity improvement is not subject to diminishing returns; Learning is *unbounded*.

- It is also assumed that learning capacity, δ_j, might differ across sectors, but not across countries.

- Furthermore, it is assumed that these learning effects are external to competitive producers that generate them. Thus, the firms do not take into account these learning effects when making production decisions.

- This allows us to solve for the dynamics by first solving for the static equilibrium at each t, holding the labor productivity as given, and look at the sequence of the static equilibriums, as the labor productivity evolves over time.

To keep it simple, let us $a_j = a_j^* = 1$ for j = 1 and 2.

Autarky Case:

Statics: Each period, the Home autarky static equilibrium is characterized by

$$X_1 = C_1 = \alpha A_1 L; \quad X_2 = C_2 = (1-\alpha) A_2 L; \qquad Y = (A_1)^\alpha (A_2)^{1-\alpha} L.$$

Furthermore, by taking the final good as the numeraire, $w = (A_1)^\alpha (A_2)^{1-\alpha}$.
Likewise for Foreign.

Exercise: Show the above.

Dynamics: from $\dot{A}_j(t) = \delta_j X_j(t)$, $\dot{A}_1(t)/A_1(t) = \delta_1 \alpha L$, $\dot{A}_2(t)/A_2(t) = \delta_2(1-\alpha)L$, and

$$g_Y \equiv \frac{\dot{Y}(t)}{Y(t)} = g_w \equiv \frac{\dot{w}(t)}{w(t)} = [\delta_1(\alpha)^2 + \delta_2(1-\alpha)^2]L$$

Likewise, $$g_{Y^*} \equiv \frac{\dot{Y}^*(t)}{Y^*(t)} = g_{w^*} \equiv \frac{\dot{w}^*(t)}{w^*(t)} = [\delta_1(\alpha)^2 + \delta_2(1-\alpha)^2]L^*$$

Note: The larger country grows faster.

Free Trade Equilibrium: Suppose that the initial labor productivities are such that

$$\frac{A_1(0)}{A_1^*(0)} > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(0)}{A_2^*(0)}.$$

Then, the static free trade equilibrium is characterized by Home specializing in Input 1 and Foreign specializing in Input 2:

$$X_1 = A_1 L; \qquad X_2 = 0; \qquad X_1^* = 0; \qquad X_2^* = A_2^* L^*$$

$$\frac{w}{A_1} = p_1 < \frac{w^*}{A_1^*}; \qquad \frac{w}{A_2} > p_2 = \frac{w^*}{A_2^*}; \qquad \frac{A_1}{A_1^*} > \frac{w}{w^*} = \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2}{A_2^*}$$

and

$$Y = wL = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} (A_1 L)^{\alpha} \left(A_2^* L^*\right)^{1-\alpha}; \qquad Y^* = w^* L^* = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (A_1 L)^{\alpha} \left(A_2^* L^*\right)^{1-\alpha}$$

Again, the final good is chosen as the numeraire, which is permissible because, even though the final good is nontradable, its price is the same in the two countries under free trade.

Exercise: Show the above. Show that the outputs of the final good go up in both countries by moving from Autarky to Free Trade.

Dynamics: From $X_1 = A_1 L$; $X_2 = 0$; $X_1^* = 0$; and $X_2^* = A_2^* L^*$,

$$\frac{\dot{A}_1(t)}{A_1(t)} = \delta_1 L; \quad \frac{\dot{A}_2(t)}{A_2(t)} = 0; \quad \frac{\dot{A}_1^*(t)}{A_1^*(t)} = 0; \quad \frac{\dot{A}_2^*(t)}{A_2^*(t)} = \delta_2 L^*.$$

This implies that the initial patterns of trade are sustained forever:

$$\frac{A_1(t)}{A_1^*(t)} = \frac{A_1(0)}{A_1^*(0)} \exp(\delta_1 L t) > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(t)}{A_2^*(t)} = \frac{A_2(0)}{A_2^*(0)} \exp(-\delta_2 L^* t) \text{ for all t} > 0.$$

Furthermore,

$$g_Y = g_w = g_{Y*} = g_{w*} = \alpha\delta_1 L + (1-\alpha)\delta_2 L^*.$$

Notes:

- More generally, Home (Foreign) specializes in 1 (2) *forever* if the initial patterns are

$$\frac{A_1(0)/a_1}{A_1^*(0)/a_1^*} > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(0)/a_2}{A_2^*(0)a_2^*}.$$

  Thus, the initial condition, or "history," matters. This could occur even when Foreign (Home) has the *natural* comparative advantage in sector 1 (2), $a_1^*/a_1 < a_2^*/a_2$.

- Both countries grow at the same rate under Free Trade. Cobb-Douglas is essential for this result, as it implies that the output growth in each sector is exactly offset by the terms of trade change. For the CES case, see the next example adopted from Lucas (1988; section 5) or Acemoglu growth manuscript, Ch.19.7.

- When the two countries are of the equal size, the (identical) growth rate goes up under Free Trade. When the two countries differ in size, the growth rate always goes up for the smaller country, but may goes down for the larger country if it ends up specializing in Sector 1, for the case, $\alpha\delta_1 < (1-\alpha)\delta_2$, or in Sector 2, for the case, $\alpha\delta_1 > (1-\alpha)\delta_2$.

Exercise: Demonstrate the above.

Exercise:

Extend the above analysis for the case where the final goods use more than two tradeable intermediate inputs with Cobb-Douglas.

Case of the CES final goods production: adopted from Lucas (1988, sec.5).

- CES final goods production, with σ is the elasticity of substitution between Input 1 & 2.
- A continuum of countries of the same size, indexed by c(t) ∈ [0,1] so that $A_1^c(t)/A_2^c(t)$ is strictly decreasing in c(t) ∈ [0,1].
- All countries in c(t) ∈ [0, m(t)) specialize in input 1; all countries in c(t) ∈ (m(t), 1] specialize in 2, where m(t) is the marginal country at t.
- All countries specializing in input-j improve its labor productivity in sector j at the rate equal to $\dot{A}_j^c(t)/A_j^c(t) = \delta_j$. The output also grows at the same rate.

Exercises:

- Show that c(t) is time-invariant; there is no need to change the country indices over time.
- Under which condition, m(t) is also time-invariant?
- Compare the output growth of different countries.

Natural Resource Abundance and Industrial Development: Matsuyama (1992)

Two Sectors: Manufacturing and Agriculture (or Natural Resource), with diminishing returns in labor, as in the Ricardo-Viner Model:

$$X_M(t) = M(t)F(n(t)); \quad F(0)=0; F'>0; F''<0$$
$$X_A(t) = AG(1-n(t)); \quad G(0)=0; G'>0; G''<0.$$

where n(t) is the employment share of the M-sector.

Learning: Agricultural productivity, A, is time-invariant, while Manufacturing Productivity, $M(t)$, evolves as $\dot{M}(t) = \delta_M X_M(t)$.

Preferences: $\beta \log(C_A(t) - \gamma) + \log(C_M(t))$, $\gamma > 0$.

Exercises:

- What is the growth effect of a higher A in Autarky?
- Suppose that the world consists of a continuum of countries, all of which share the same A and $M(t)$. Suppose that one country experiences a temporary or permanent increase in A. What are the dynamic effects of these changes on this economy?

Note: Rodriguez-Rodrik (2000) extended the above model to look at the effects of import tax, and used that model to organize a critical review of cross-country empirical studies on the effects of growth and trade.

Unbounded LBD that is Country-Specific, but not Sector Specific:

Let us modify the above model as follows.

- **A Continuum of Small Countries**, indexed by $c \in [0,1]$.
- **Two Sectors;** Manufacturing and Agriculture, each subject to diminishing returns in labor: $X_M^c(t) = A^c(t)F^c(n^c(t))$; $X_A^c(t) = A^c(t)G^c(1-n^c(t))$.

Notes:

- ➢The two sectors share the same productivity parameter within each country.
- ➢F^c and G^c depend on *c,* to capture the natural comparative advantage.

- **LBD:** Only the M-Sector is the engine of growth, whose activities generate learning in both sectors. Learning is, however, country-specific: $\dot{A}^c(t) = \delta_M X_M^c(t)$.

Exercise: Analyze the impact of trade on the growth of each country.

Unbounded LBD that is Sector-Specific, but only partially Country-Specific:

Krugman's (1987) Model with Two Countries and a Continuum of Goods:

- Let us modify the DFS (1977) Ricardian Model with the following LBD.

$$\dot{A}_z(t) = \delta[X_z(t) + \rho X_z^*(t)]; \qquad \dot{A}_z^*(t) = \delta[\rho X_z(t) + X_z^*(t)], \qquad z \in [0,1].$$

where $A_z(t)$ and $A_z^*(t)$ are the Home and Foreign Labor Productivity in Sector z at time t. LBD is confined to each sector. With $0 < \rho < 1$, there are some cross-country spillovers.

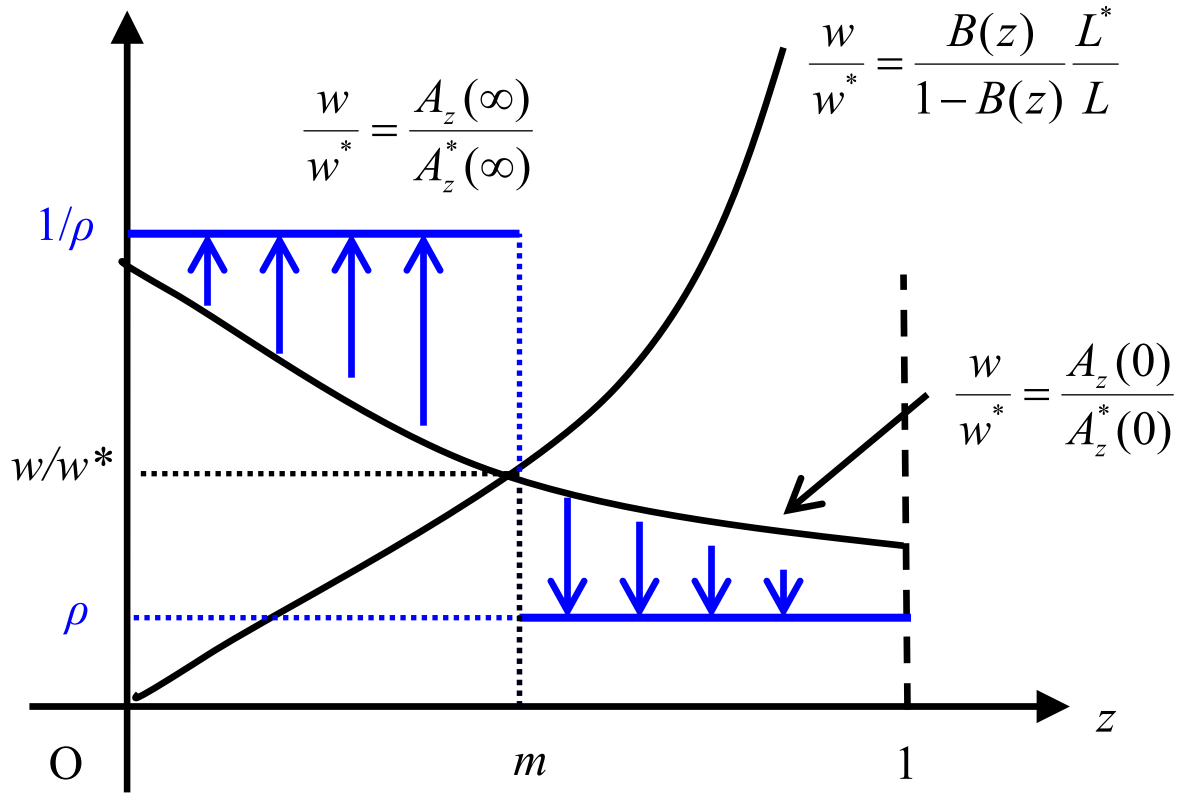
- Again, let us index the sectors so that the Home relative labor productivity, $A_z(t)/A_z^*(t)$, is decreasing in $z \in [0,1]$. Then,

$$\dot{\left(\frac{A_z}{A_z^*}\right)} = \delta L_z\left(1 - \rho\frac{A_z}{A_z^*}\right)\left(\frac{A_z}{A_z^*}\right) \qquad \text{for } z \in [0,m).$$

$$\dot{\left(\frac{A_z}{A_z^*}\right)} = \delta L_z^*\left(\rho - \frac{A_z}{A_z^*}\right) \qquad \text{for } z \in (m,1].$$

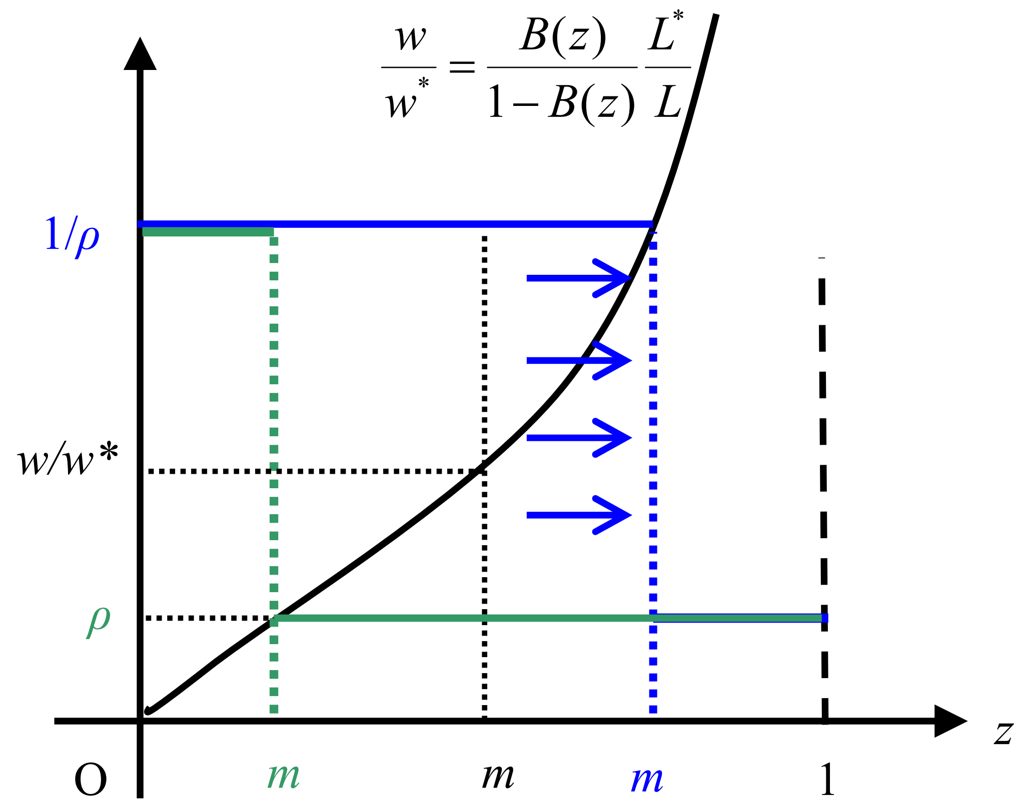
- If $\rho < w/w^* = A_m(0)/A_m^*(0) < 1/\rho$, the patterns of trade never change, and the relative labor productivity converges to

$$\frac{A_z(t)}{A_z^*(t)} \to \frac{1}{\rho} \text{ for } z \in [0,m); \qquad \frac{A_z(t)}{A_z^*(t)} \to \rho \quad \text{for } z \in (m,1].$$



Notice ***a multiplicity of steady states***, depending on the initial condition. ***History matters.*** If the initial patterns of trade are such that $m \in [m, m]$, the steady state ToT is determined by $B(m)L^*/(1\text{–}B(m))L$. The steady state Home welfare would be higher with a higher m.

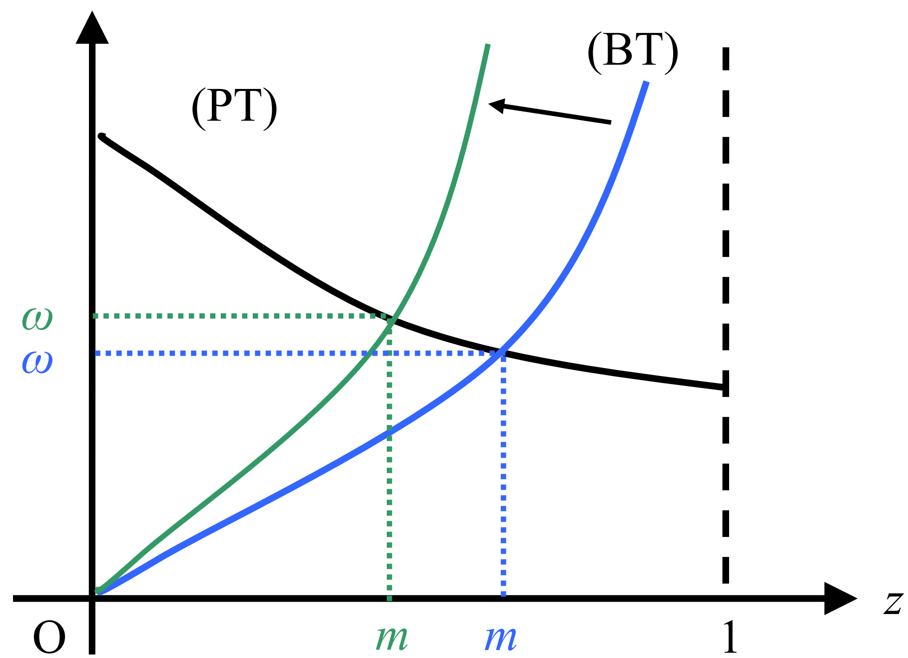
- Home has an incentive to subsidize (temporarily) a few sectors slightly above the marginal sector, so that they can gain experiences and take over.
- This expands the range of goods it produces, increases the demand for its labor, and improves its ToT.
- This incentive to slice off a few sectors near the margin at a time continues until the ToT reaches to the upper limit. (Krugman called this "**narrow moving band**.")
- Obviously, Foreign has the same incentive to do so.



Krugman (1987) also explored similar mechanisms in the DFS Ricardian model with nontradeable goods.

Recall that, in that model, a temporary transfer from Foreign to Home causes the Home real exchange rate to appreciate, as shown in the figure. As the Home labor becomes more costly, some industries move from Home to Foreign.

- This allows the Foreign sectors near the margin of competitiveness to gain experiences and take over.
- When the transfer is over, the patterns of trade may shift permanently to Foreign.
- In the long run, Home (Foreign) may lose (benefit) from receiving (giving) the temporary transfer.
- Krugman also discussed the Dutch Disease and the long run effects of a temporarily tight monetary policy ("**Competitive Consequences of Mrs. Thatcher**.")



Yanagawa's (1996) Two-Sector Model with a Continuum of Countries
(Unfinished)

Bounded Learning:

- We have so far assumed that unbounded productivity growth is possible through LBD.
- Empirical evidence from any particular manufacturing activity suggests strong diminishing returns to LBD.
- Even if we interpret LBD more broadly as a reduced form way of modeling the causality from the industry size to its productivity gains, it is hard to imagine how productivity could grow unbounded in any particular industry.
- However, even if productivity growth is bounded in each industry, the economy may be able to sustain long run growth if experiences in one industry helps the economy to move into more sophisticated, higher value-added activities.
- At the same time, bounded learning also suggests the possibility that having expertise in certain areas, the economy may fail to move into new, more promising activities.
- To see this, let us imagine that the M-sector consists of many industries.

***Learning-By-Doing Model with Many Industries* (j = 1, 2,…,J)**

$X_t^j = A^j(Q_t^j)L_t^j$, where X_t^j: Output in Industry j; L_t^j: Employment in Industry *j*.

$A^j(Q_t^j)$; Labor Productivity, increasing in Q_t^j, the cumulative experience in *j*.

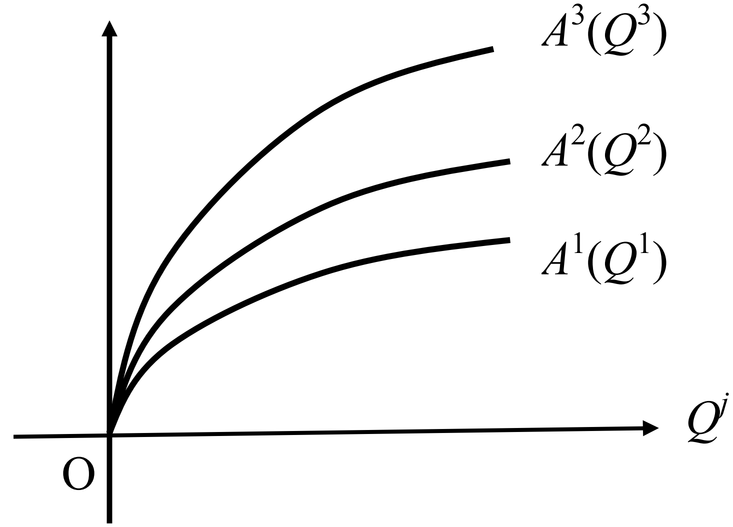
The state space is J-dimensional; $\mathbf{Q_t} \equiv [Q_t^j]$.

All manufacturing goods, $j = 1, 2, \ldots, J$, are ***Perfect Substitutes***. Or, the prices are exogenously determined in the world market and normalized to one.

$$X_t^M = \sum_{j=1}^{J} X_t^j = \sum_{j=1}^{J} A^j(Q_t^j)L_t^j.$$

$$\Rightarrow X_t^j = A^j(Q_t^j)L_t^j > 0 \quad \text{only if } w_t = A^j(Q_t^j) = Max_k\{A^k(Q_t^k)\}.$$

- $A^j(q) = \lambda^{j-1} A(q)$, with $\lambda > 1$;
  $A(q)$ is strictly increasing and $A(0) = 0$.



Note: Higher-indexed goods are *potentially* more productive. Or we may think higher-indexed goods are of higher quality, as follows:

$$X_t^M = \sum_{j=1}^{J} \lambda^{j-1} X_t^j \text{ with } A^j(q) = A(q).$$

- *No Inter-industry spillovers*; each industry learns only from its own production.

$$\dot{Q}_t^j = \delta_j \left(L_t^j - Q_t^j \right) \text{ or } Q_t^j = Q_0^j \exp(-\delta_j t) + \int_0^t Q_0^j \exp[\delta_j (s-t)] ds$$

Note: Here, knowledge is assumed to depreciate. That is, you could also "forget by not-doing." This prevents Q and A(Q) from growing forever. Alternatively, we could have assumed A(Q) is bounded above, in which we could let Q grow unbounded. Notice that the depreciating rate is set equal to the learning speed. No loss of generality, here. *Why?*

- Furthermore, suppose, for simplicity, the total manufacturing employment is fixed at L^M.

$$L^M = \sum_{j=1}^{J} L_t^j .$$

Then, it is easy to see that there are J-stable steady states: $\mathbf{Q} = (0, 0, \ldots, L^M, 0, \ldots, 0)$.

Exercise: Why do I need the assumption, $A(0) = 0$? How would you change the specification if you want to keep the same conclusion with $A(0) > 0$?

Now, let us see what might happen if there are some inter-industry learning spillovers?

Unbounded Growth through Bounded Learning

Suppose J = 2 and

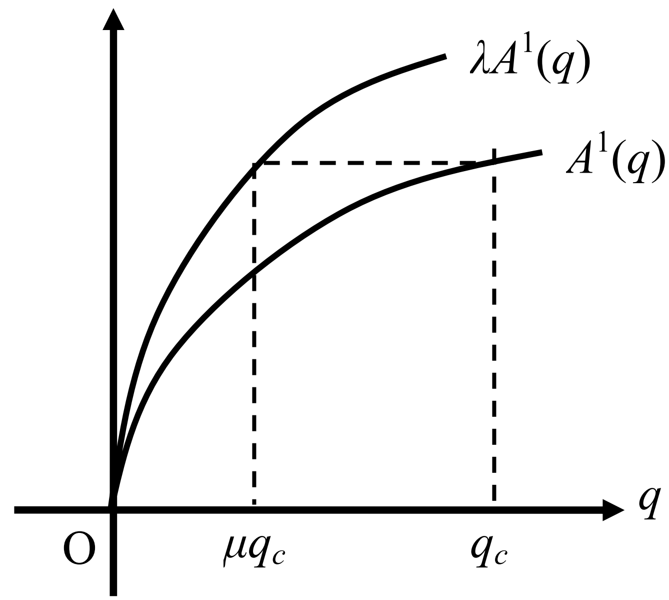
$$X_t^1 = A(Q_t^1)L_t^1;$$

$$X_t^2 = \lambda A(\mu Q_t^1 + Q_t^2)L_t^2$$

with $\mu < 1 < \lambda$. Assume further:

- $\lambda A(\mu q)/A(q)$ is strictly increasing in q.
- $\lambda A(\mu q_c)/A(q_c) = 1$.

e.g., $A(q) = q/(q+\alpha)$ with $\lambda\mu < 1 < \lambda$.
q_c is decreasing in λ and in μ.



Starting from the initial condition, $0 < Q_t^1 < q_c$ and $Q_t^2 = 0$,

For $L^M < q_c$, the economy is trapped in Industry 1.

For $L^M > q_c$, the economy makes a successful transition from Industry 1 to 2.

A larger L^M, a larger λ and a larger μ can the successful transition more likely.

This idea can be extended for the case of a countably infinite number of industries:

$$L^M = \sum L_t^j = \sum \frac{X_t^j}{\lambda^j A(\mu Q_t^{j-1} + Q_t^j)}$$

For $L^M < q_c$, the economy is trapped in Industry 1.
For $L^M > q_c$, the economy makes a successive transition from 1 to 2, to 3, to 4, …., and its labor productivity grows indefinitely.

- Stokey (1988) illustrated this idea in a closed economy model with a continuum of goods, where the goods are *imperfect* substitutes. (They are vertically differentiated in the manner discussed in Part II). At any time in point, the economy produces a finite range of goods, which moves up over time.

- Young (1991) explored this idea in a two-country Ricardian model with a continuum of goods, where the goods are *imperfect* substitutes (but horizontally differentiated). Two countries produce (non-overlapping) finite ranges of goods. The the country with more experienced grow faster, while the other country might be worse off under Free Trade.

- For more on this issue, see Lucas (1993) "Making a Miracle" paper.

Inter-Industry Spillovers of Industry-Specific Bounded Learning through Demand Effects:

Matsuyama (2002) shows that, even if knowledge spillovers are confined within each industry, productivity gains can spillover to other industries through the Demand effects.

Closed Economy Model with J-goods:

- Each good comes in discrete units.
- They are ordered by the priority (or hierarchical needs), similar to Matsuyama (2000) Ricardian model.

Industry-Specific LBD:

$$X_t^j = A^j(Q_t^j)L_t^j; \qquad \dot{Q}_t^j = \delta_j\left(L_t^j - Q_t^j\right) \qquad (j = 1, 2, \ldots, J).$$

Taking labor as the numeraire, the price of good-j is $A^j(Q_t^j)$. .

Heterogeneous Households (with a measure N**):** They differ in the (effective units of) labor endowment:

$F(\bullet)$: the cdf of the labor endowment, and hence the household purchasing power.

Individual Demands: Because of hierarchical preferences, each household buys one unit of Good 1 first, then one unit of Good 2, and then one unit of Good 3, and so on, as long as they can afford. The richer households buy all the goods the poor households buy, and some more.

Aggregate Demand: Since only the households whose income exceed $\sum_{k=1}^{j} A^k(Q_t^k)$ buy good-j, and no household buys more than one unit of good-j, the total demand for good-j is

$$D^j(Q_t) = N\left[1 - F\left(\sum_{k=1}^{j} A^k(Q_t^k)\right)\right]$$

Dynamics: $\dot{Q}_t^j = \delta_j\left(D_t^j(Q_t) - Q_t^j\right) = \delta_j\left\{N\left[1 - F\left(\sum_{k=1}^{j} A^k(Q_t^k)\right)\right] - Q_t^j\right\}$

- This system is cooperative (in the sense of Hirsch), but in an asymmetric way. Productivity gains in industry j help all industries, $k > j$, and not industries, $k < j$.

- No learning or knowledge spillovers are present in this model. Inter-industry spillovers are entirely due to demand complementarity.

- The set of steady states is a *lattice*; it depends sensitively on F. Thus, small differences in the income distributions could make big differences.

Matsuyama (2002) used this model to study:

- the mechanism behind the Rise of Mass Consumption Societies, where the economy grows as an increasingly large number of households enjoys an increasingly large number of goods.
- Flying Geese Patterns, where different industries take off one after another.
- Why some societies fail to make that transition.

Its open economy implications are yet to be explored.

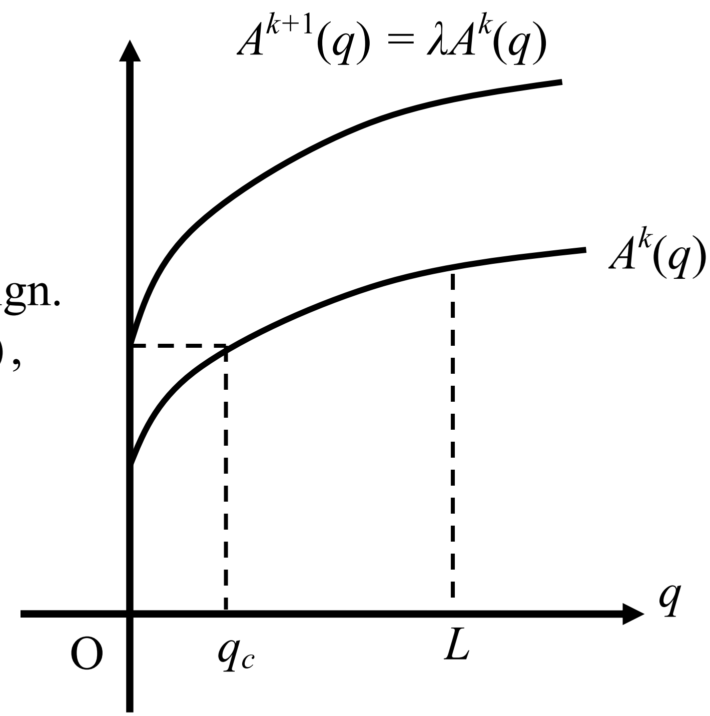
Bounded Learning and Leapfrogging: based on Bresiz-Krugman-Tsiddon (1993)

- Many trade models with LBD imply that a technologically more advanced country has advantage in achieving higher growth, often at the expense of countries behind.
- But, we have observed changes in the technological leader, from the Italian city states, to the Dutch Republic, to Great Britain and to the US.
- The following model, adopted from Bresiz et. al. (1993), suggests that this may be a natural consequence of the bounded LBD.

Two Countries: Home and Foreign, the equal size, $L = L^*$. It is assumed that $L = L^*$ is sufficiently large.

Two (Tradeable) Sectors: Agriculture & Manufacturing.

- Agriculture produces a homogenous good. We index it by 0.
- Manufacturing consists of many (perfectly substitutable) goods, indexed by j = 1, 2, …, We assume that only the first *k* goods are initially available and study the impact of an exogenous arrival of the (k+1)-th generation of the good.



Preferences: $\left(C_t^0\right)^{1-\beta}\left(C_t^M\right)^{\beta} = \left(C_t^0\right)^{1-\beta}\left(\sum_j C_t^j\right)^{\beta}$.

Technologies:

- Agricultural labor productivity is one at Home & Foreign.
- Labor productivity of Industry j is $A^j(Q_t^j) = \lambda^{j-1}A(Q_t^j)$,

with $\lambda > 1$, where $A(\bullet)$ is strictly increasing but bounded and satisfies:

$$\frac{A(L)}{A(0)} > \frac{\beta}{1-\beta} > \frac{A(L)}{\lambda A(0)} > 1$$

and

$$\dot{Q}_t^j = \delta_j(L_t^j - Q_t^j), \text{ with } Q_t^j \le L$$

Note: The above inequalities requires $A(0) > 0$. The assumption that $A(\bullet)$ is bounded is not necessary, but, since L is assumed to be sufficiently large, this makes it easier to satisfy the above inequalities.

Let us now demonstrate that, under these assumptions,

❖ ***Once Home specializes in the k-th M-good, and Foreign specializes in A-good, it is a steady state.***

1) As long as Home specializes in (any) M-good and Foreign specializes in A-good, the Balanced Trade condition also requires that $\beta w_t^* L^* = (1-\beta) w_t L$, or $\frac{w_t}{w_t^*} = \frac{\beta}{1-\beta}$.

2) The condition that Home specializes in k and Foreign specializes in 0 at T_0 is given by

$$\frac{\lambda^{k-1} A(Q_{T_0}^k)}{Max_{1\le j\le k}\{\lambda^{j-1} A(Q_{T_0}^{*j})\}} > \frac{w_{T_0}}{w_{T_0}^*} = \frac{\beta}{1-\beta} > 1.$$

3) Furthermore, since $X_t^j = 0$ for all $1 \le j < k$, $X_t^k > 0$, and $X_t^{*j} = 0$ for all $1 \le j \le k$,

$\dot{Q}_t^j \le 0$ for all $1 \le j < k$, $\dot{Q}_t^k \ge 0$, and $\dot{Q}_t^{*j} \le 0$ for all $1 \le j \le k$

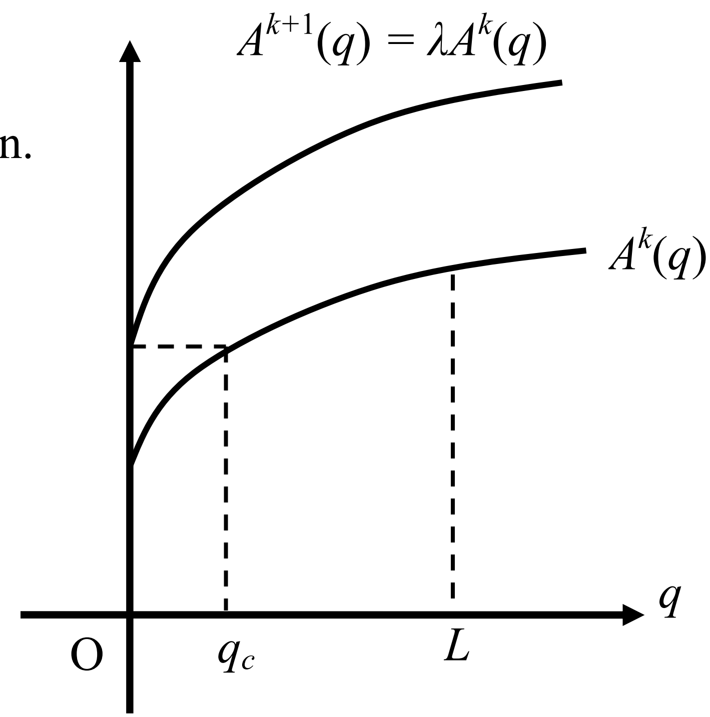
$$\Rightarrow \frac{\lambda^{k-1} A(Q_t^k)}{Max_{1\le j\le k}\{\lambda^{j-1} A(Q_t^{*j})\}} > \frac{w_t}{w_t^*} = \frac{\beta}{1-\beta} > 1 \text{ for all } t \ge T_0.$$

Thus, Home specializes in k and Foreign specializes in 0 for all $t \ge T_0$.

Suppose that the world economy has been in this situation long enough that Q_t^k has grown sufficiently close to L and Q_t^{*k} has shrunk sufficiently close to zero. Then, a new generation of the M-good, k+1, arrives at T_1. What happens?

1) Labor productivity for the new good starts at $A^{k+1}(0) = A^k(q_c)$, where $0 < q_c < L$. Thus, $A^k(Q_t^k) > A^{k+1}(0) > A^k(Q_t^{*k})$.

The k-th is more productive than the (k+1)-th at Home.
The (k+1)-th is more productive than the k-th at Foreign.



2) $\dfrac{\beta}{1-\beta} > \dfrac{A^k(Q_t^k)}{A^{k+1}(0)} \approx \dfrac{A(L)}{\lambda A(0)} > 1$ implies that the (k+1)-th industry at Foreign, whose wage is lower, can compete with the k-th industry at Home. Thus, from T_1 on, Foreign starts producing both 0 and k+1, while Home continues to specialize in k, and the relative wage satisfies

$$\frac{\beta}{1-\beta} > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} > 1.$$

3) As Foreign improves its productivity in k+1 (faster than Home does in k), both countries become equally efficiency at T_2. After T_2, the patterns of comparative advantage are reversed. Foreign has CA in manufacturing, and starts specializing k+1, while Home has CA in Agriculture, and produces both 0 and k, with

$$1 > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} > \frac{1-\beta}{\beta}.$$

4) This could continue until the relative manufacturing efficiency reaches to the point where Home stops producing k and starting specializing in 0, with

$$1 > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} = \frac{1-\beta}{\beta}.$$

5) This situation continues as a new steady state, at least until the (k+2)-th generation of the good arrives.

In summary,

The lagging country, due to its *lack* of expertise in the existing technology, has a *comparative* advantage in the new technology, making it possible to take over the technology leader.

Growth and Trade: Factor Accumulation

In Part I, we examined the effects of exogenous changes in the factor endowments in static competitive models of trade (e.g., Bhagwati's immiserizing growth)

While they are useful, we now want to look at the roles of factor accumulations in dynamic trade models, at least for two reasons.

1) Static Models treat the factor endowments exogenous. Many factors, particularly capital (whether physical or human), can be accumulated. It is thus unsatisfactory to treat the factor proportions differences, such as the capital-labor ratio, as the cause of trade. We may want to look at more "fundamental" causes or "deep parameters" behind these differences.

2) We need to understand how international trade affects growth and development processes. This is important because much of growth theory is developed under the closed economy assumption, and, in spite of that, its implications are tested with the cross-country data, even though no country is in autarky in reality.

One-Page Refresher on One-Sector, Closed Economy, Growth Models

Resource Constraint: $\dot{K}_t = R(Y_t - C_t) - \delta K_t = sRY_t - \delta K_t$,

s: Aggregate saving rate
R: Productivity of investment technologies
(or the inverse measure of the investment distortions)

Harrod-Domar (AK) Model: $Y_t = F(K_t) = ZK_t$; Z: Total Factor Productivity (TFP)

$$\Rightarrow \dot{K}_t = sRZK_t - \delta K_t \qquad \Rightarrow g_t = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sRZ - \delta.$$

Message: *With the linear accumulation technology, the long run growth rate depends on the saving rate, s, the investment productivity, R, and the production productivity Z.*

Solow (Neoclassical) Model: $Y/K = ZF(K)/K$ is decreasing in K and $\lim_{K\to\infty}F(K)/K = 0$.

$$\Rightarrow \frac{\dot{K}_t}{K_t} = \frac{sRZF(K_t)}{K_t} - \delta \to \frac{sRZF(K_\infty)}{K_\infty} - \delta \equiv 0, \qquad \Rightarrow g_\infty = 0 \text{ at } Y_\infty = ZF(K_\infty).$$

Message: *With the accumulation technology subject to diminishing returns, s, R, and Z, change Y_∞ but not g_∞. They have "level" effects, but no "growth" effect in the long run.*

Factor Accumulation in a Ricardian Model of Trade

The following is inspired by Acemoglu and Ventura (2002).

Two Countries: Home and Foreign(*)

One Primary Factor of Production (Capital): K_t and K_t^* (the factor prices, r and r^*)
Reproducible but nontradeable.

A Continuum of (Tradeable) Intermediate Input Sectors: $z \in [0,1]$.

Unit Capital Requirements: $a(z)$, $a^*(z)$, with strictly decreasing $A(z) = a^*(z)/a(z)$.

(Nontradeable) Final Good Sector: Cobb-Douglas Technologies:

$$\log Y_t = \int_0^1 \log(c_t(z))dz; \qquad \log Y_t^* = \int_0^1 \log(c_t^*(z))dz$$

$c_t(z)$ ($c_t^*(z)$): input z used in the final good production at Home (Foreign).

Unit Cost Functions of the Final Good:

$$\exp[\int_0^1 \log(p(z))dz], \qquad \exp[\int_0^1 \log(p^*(z))dz],$$

$p_t(z)$ ($p_t^*(z)$): the prices of input z

Capital Accumulation:

- Final Goods may be consumed or invested to accumulate capital.

- Representative agents consume a constant fraction of the final good.

$$\dot{K}_t = R[Y_t - C_t] = sRY_t; \qquad \dot{K}_t^* = R^*[Y_t^* - C_t^*] = s^*R^*Y_t^*$$

where s and s^* are the saving rates and R and R^* are the efficiencies of the investment technologies (or the inverse measure of the investment distortions).

Home Autarky Equilibrium: taking the final good as the numeraire,

$$1 = \exp[\int_0^1 \log(p_t(z))dz] = \exp[\int_0^1 \log(a(z)r_t)dz] \qquad \Rightarrow r_t = \exp[-\int_0^1 \log(a(z))dz] \equiv Z,$$

$$\log Y_t = \int_0^1 \log(K_t / a(z))dz \Rightarrow Y_t = r_t K_t = ZK_t \Rightarrow \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sR\frac{Y_t}{K_t} = sRZ \equiv g^A.$$

Foreign Autarky Equilibrium: Likewise,

$$r_t^* = \exp[-\int_0^1 \log(a^*(z))dz] \equiv Z^*; \qquad \frac{\dot{Y}_t^*}{Y_t^*} = \frac{\dot{K}_t^*}{K_t^*} = s^* R^* Z^* \equiv g^{*A},$$

Z and Z^*: TFP

Note: in Autarky, both countries grow as in the Harrod-Domar (*AK*) model.

Trade Equilibrium:

The final good prices are equalized. Again, taking the final good as the numeraire,

$$\int_0^1 \log(p_t(z))dz = \int_0^{m_t} \log(a(z)r_t)dz + \int_{m_t}^1 \log(a^*(z)r_t^*)dz = 0.$$

Using the Patterns of Trade (PT) condition,

(PT): $$\frac{r_t}{r_t^*} = A(m_t) \equiv \frac{a^*(m_t)}{a(m_t)},$$

this can be written to:

$$r_t = Z\exp\left[\int_{m_t}^1 \log\left(\frac{A(m_t)}{A(z)}\right)dz\right] \equiv ZW(m_t) > Z;$$

$$r_t^* = Z^*\exp\left[\int_0^{m_t} \log\left(\frac{A(z)}{A(m_t)}\right)dz\right] \equiv Z^*W^*(m_t) > Z^*,$$

showing the (static) gains from trade.

Terms of Trade and Patterns of Trade:

At any *t*, K_t / K_t^* is given, which determines the patterns of trade, m_t, by

(BT)+(PT): $$\frac{m_t}{1-m_t}\frac{K_t^*}{K_t} = \frac{r_t}{r_t^*} = A(m_t),$$

Dynamics: The above equation in turn determines capital accumulation as follows:

$$\frac{\dot{K}_t}{K_t} = sRr_t = sRZW(m_t); \qquad \frac{\dot{K}_t^*}{K_t^*} = s^*R^*r_t^* = s^*R^*Z^*W^*(m_t).$$

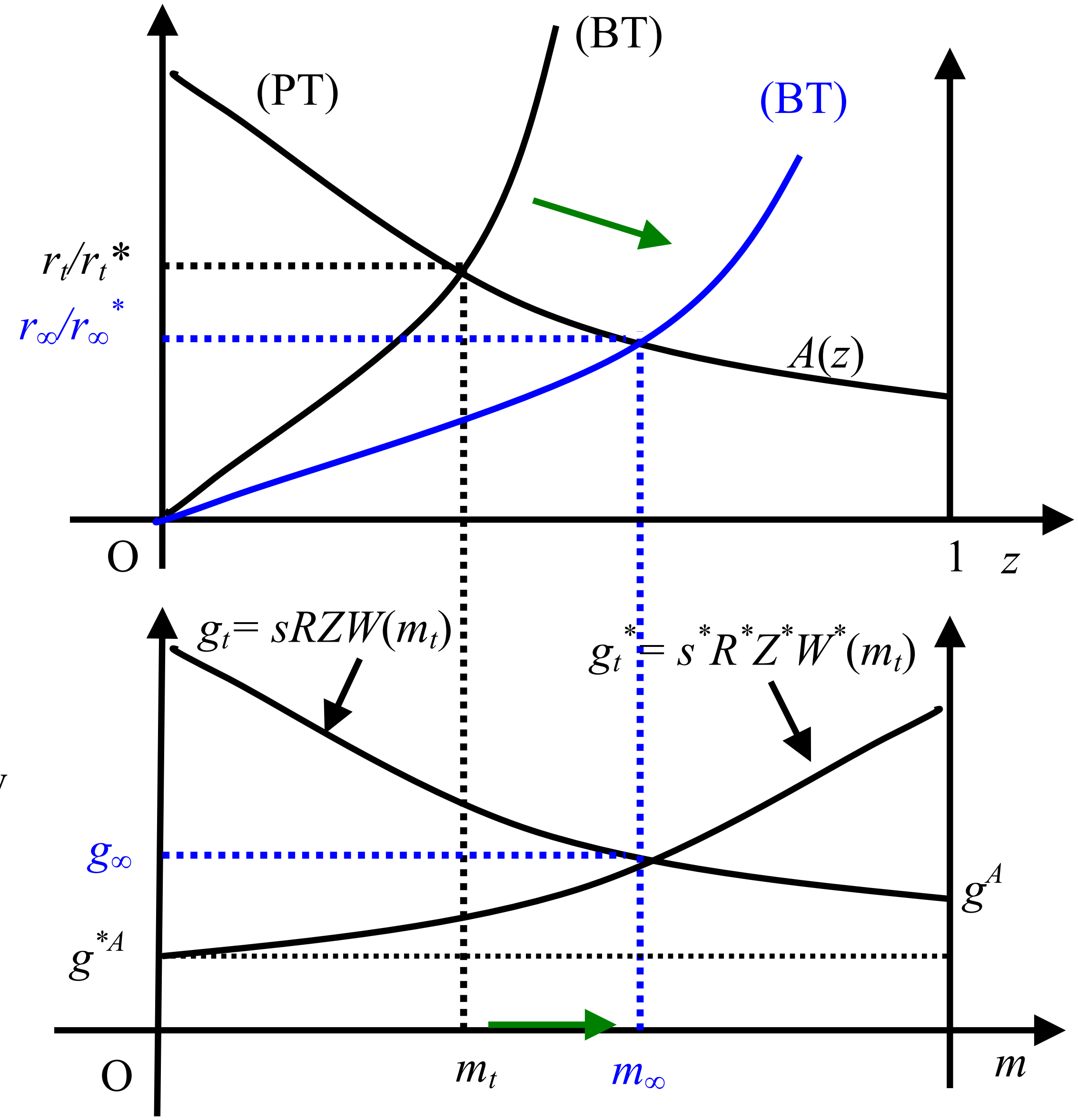
Steady State:

$$\frac{m_\infty}{1-m_\infty}\left(\frac{K^*}{K}\right)_\infty = \frac{r_\infty}{r_\infty^*} = A(m_\infty); \qquad sRZW(m_\infty) = g_\infty = s^*R^*Z^*W^*(m_\infty).$$

A Graphic Illustration:

Suppose $K_t / K_t^* < K_\infty / K_\infty^*$.

- The equilibrium in t is given by the intersection of the two black curves, (PT) & (BT) in the upper panel.
- The lower panel shows that, at this equilibrium, both countries grow faster than in autarky, and that Home accumulates capital faster than Foreign.
- This shifts the (BT) curve to the right, as shown by the Green Arrow in the upper panel.
- This process continues until the world economy reaches the steady state, depicted in Blue.



Thus, when they trade with each other,

- Both countries grow faster than in autarky: $g_t > g^A$; $g_t^* > g^{*A}$.

- In the steady state, they grow at the common rate; $g_\infty = g_\infty^*$.

- A higher s, a higher R, and a higher Z lead to a higher m_t, and a deterioration of the Home ToT. They increase the common steady state growth rate.

- Home's share in the world income is equal to m_t. Thus, a change in the parameters, s, R, and Z, has only the (relative) level effects, but no (relative) growth effects in the long run, somewhat similar to the Solow (Neoclassical) growth model.

Basic Message:

In spite of the linear accumulation technology, the endogeneity of the ToT and the patterns of trade introduce *de facto* diminishing returns in the growth process of each country. → growth convergence across countries.

Note: The above model differs from Acemoglu and Ventura (2002) in that

- AV solve for the intertemporal optimization problem by the representative agent, instead of the exogenous saving.
- AV assume that the final goods production is the Cobb-Douglas composite of the nontradeable capital and the CES aggregate of the tradeable intermediates, with the elasticity of substitution, $\varepsilon > 1$. Here, it is the Cobb-Douglas aggregate of the tradeable intermediates only.
- AV considered an arbitrary number of countries under the Armington assumption (hence, the patterns of trade do not respond to ToT changes).

But, the message is essentially the same.

Exercise: Instead of the exogenous saving rate, let us assume that the Home representative agent chooses the consumption path to maximize:

$$\int_0^{\infty} \log(C_t)\exp(-\rho t)dt \text{ subject to } \dot{K}_t = R[Y_t - C_t] = R[r_t K_t - C_t].$$

1) Show that $C_t = \rho K_t / R$, and hence $\dot{K}_t / K_t = R r_t - \rho$.
2) Suppose that the Foreign represent agent solves the same problem with the discount rate, ρ^*. Analyze the world economy equilibrium, for a given (ρ, R, Z) and (ρ^*, R^*, Z^*).

Exercise: Redo the above analysis for the final goods production technologies given by:

$$\log Y_t = (1-\tau)\log(n_t) + \tau\int_0^1 \log(c_t(z))dz\,;$$
$$\log Y_t^* = (1-\tau)\log(n_t^*) + \tau\int_0^1 \log(c_t^*(z))dz\,,$$

where n and n^* are the nontradeable intermediates, one unit of which is produced with one unit of capital.

Exercise: Read Acemoglu and Ventura (2002). Explain why AV assumed CES with $\varepsilon > 1$, instead of the Cobb-Doulgas technologies.

Factor Accumulation in the 2x2 Heckscher-Ohlin Model of Trade

A Small Open Economy: normalize the prices of all tradeables to one.

Two Primary (Nontradeable) Factors of Production:

Capital: K_t (the factor price, r_t) is reproducible.

Labor: L_t (the factor price, w_t) grows exogenously at the rate, n.

Two Competitive (Tradeable) Intermediate Inputs Sectors: No FIR

One (Nontradeable) Final Good Sector: consumed or invested to accumulate capital

Technologies: All CRS

		Production Functions	Unit Cost Functions
Intermediate Inputs Sectors	K-Intensive	$Y_t^K = F^K(K_t^K, L_t^K)$	$c^K(r_t, w_t) \geq p_t^K = 1$
	L-Intensive	$Y_t^L = F^L(K_t^L, L_t^L)$	$c^L(r_t, w_t) \geq p_t^L = 1$
Final Goods Sector		$Y_t = F(X_t^K, X_t^L)$	$c(p_t^K, p_t^L) = c(1,1) = 1$

Note: The price of the final good is c(1,1), which is set to one by normalization.

Capital Accumulation: $\dot{K}_t = R[Y_t - C_t] - \delta K_t = sRY_t - \delta K_t$

In Autarky: $Y_t^K = X_t^K$ and $Y_t^L = X_t^L$. Hence, the aggregate production function is:

$$F^A(K,L) \equiv \underset{K^K,K^L,L^K,L^L \geq 0}{Max} \left\{ F\left(F^K(K^K,L^L), F^L(K^K,L^L)\right) \Big| K^K + K^L \leq K; L^K + L^L \leq L \right\}.$$

$$\Rightarrow \dot{K}_t = sRF^A(K_t, L_t) - \delta K_t$$

$$\Rightarrow \dot{k}_t = sRf^A(k_t) - (n+\delta)k_t, \text{ where } f^A(k) \equiv F^A(k,1).$$

In Trade: $Y_t^K + Y_t^L = X_t^K + X_t^L$. Hence, the aggregate production function is:

$$F^T(K,L) \equiv$$

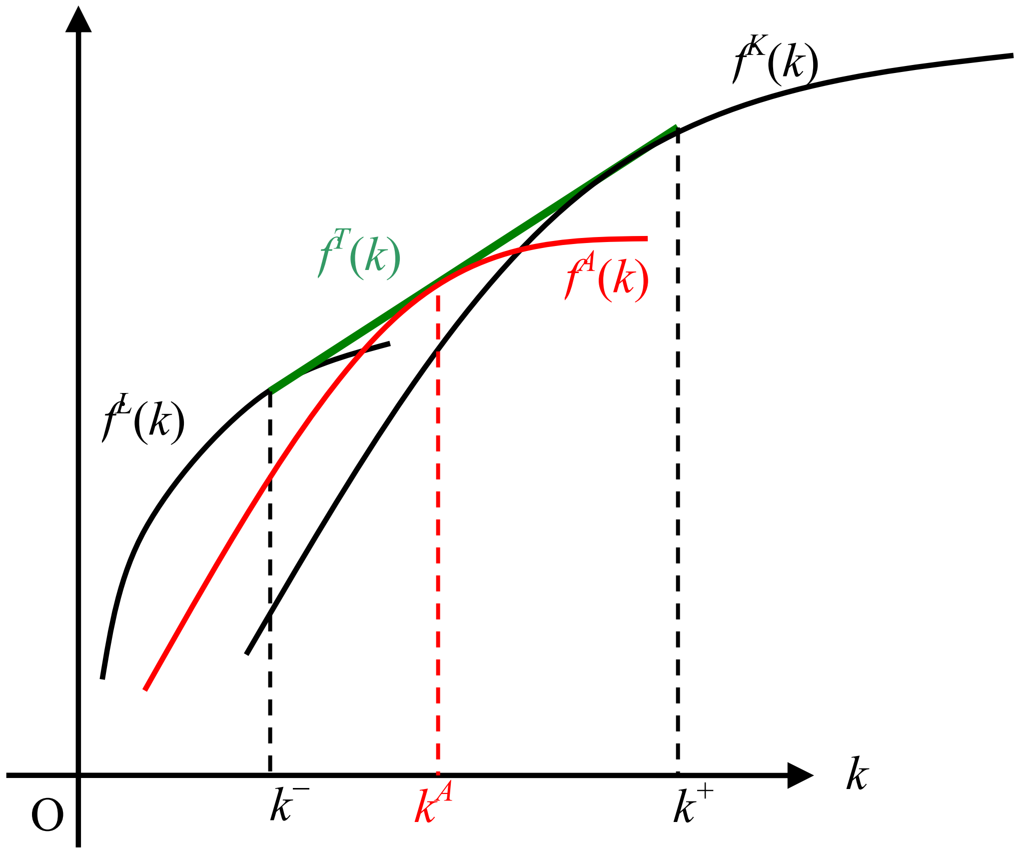
$$\underset{K^K,K^L,L^K,L^L \geq 0}{Max} \left\{ F\left(X^K, X^L\right) \Big| F^K(K^K,L^K) + F^L(K^L,L^L) = X^K + X^L; K^K + K^L \leq K, L^K + L^L \leq L \right\}$$

$$\Rightarrow \dot{K}_t = sRF^T(K_t, L_t) - \delta K_t$$

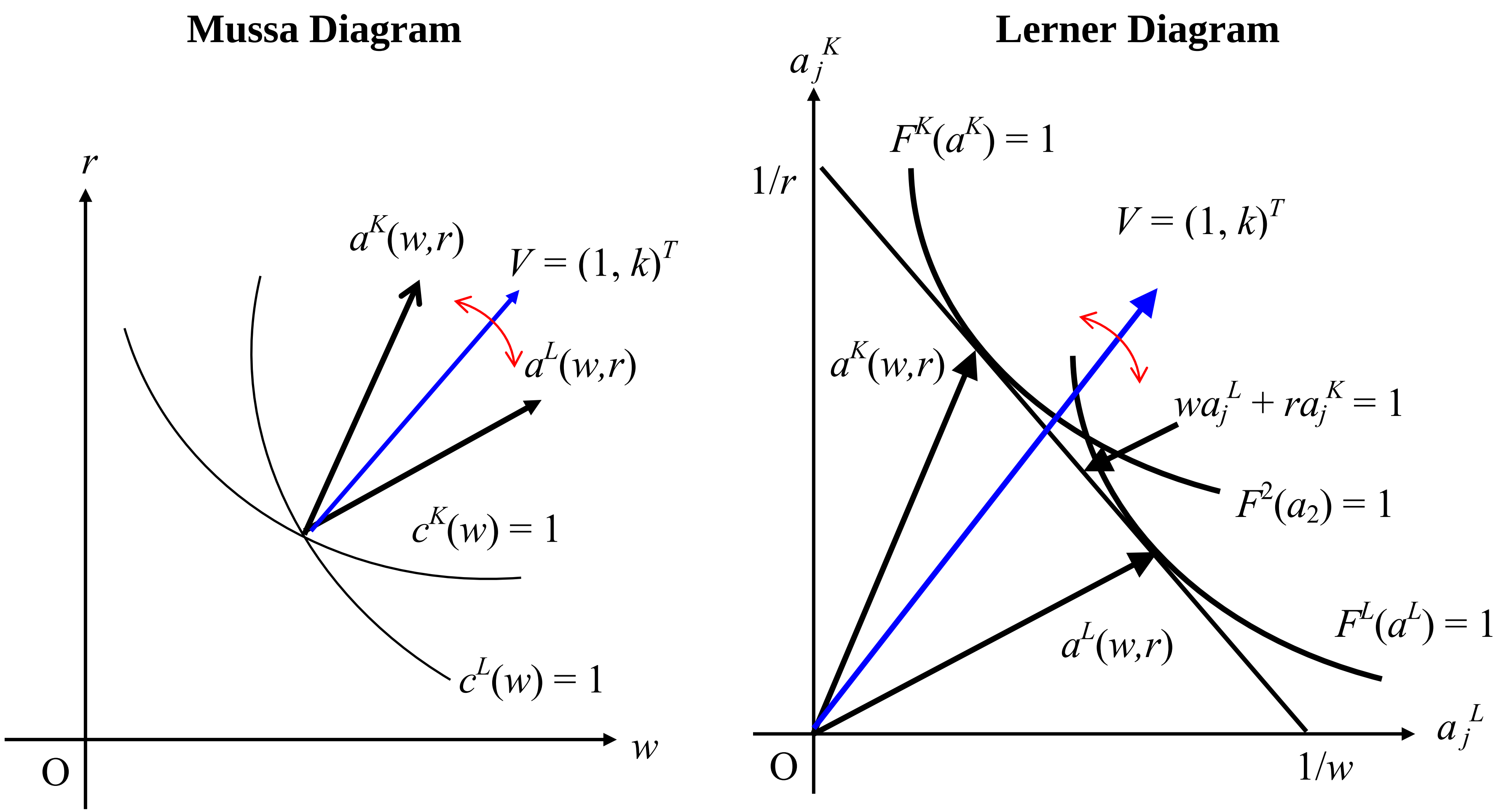
$$\Rightarrow \dot{k}_t = sRf^T(k_t) - (n+\delta)k_t, \text{ where } f^T(k) \equiv F^T(k,1).$$

Aggregate Production Functions under Autarky and under Free Trade:

- $f^T(k)$ is obtained as the upper envelope of $f^K(k)$ and $f^L(k)$, which gives its flat segment.
- $f^T(k)$ generally dominates $f^A(k)$, except one point where this country's net trade is zero.



Recall the factor price insensitivity under diversification in the 2x2 HO model.



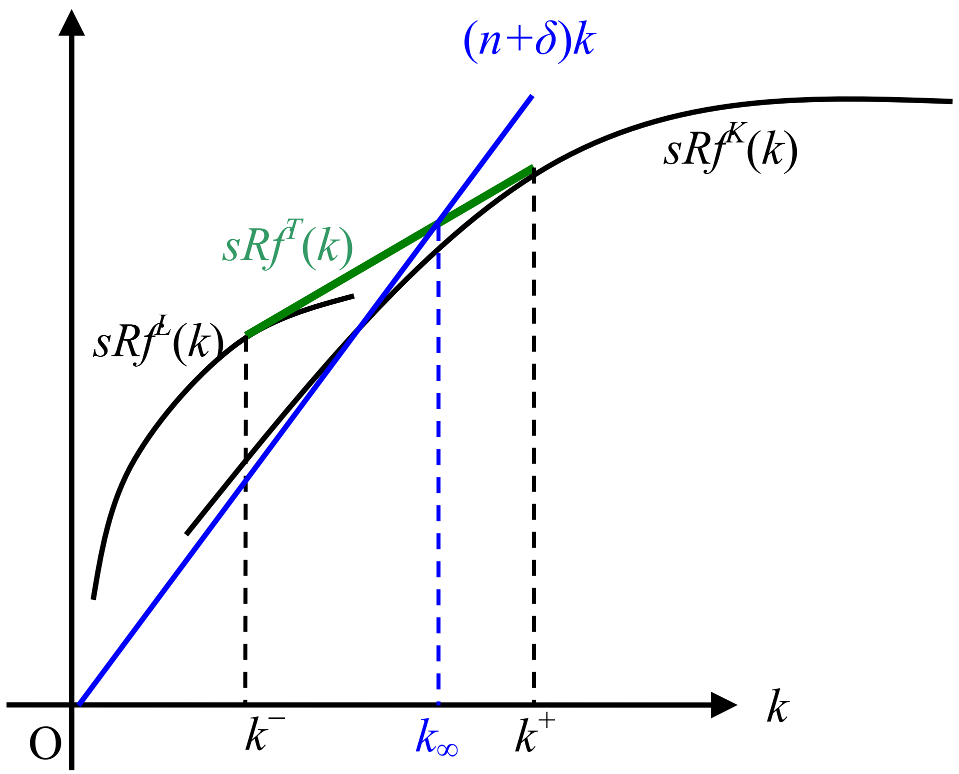
Solow Growth Diagram under Trade;

For $k_t < k^-$, the economy specializes in the labor intensive input.
For $k_t > k^+$, the economy specializes in the capital-intensive input.

For $k^- < k_t < k^+$, the economy produces both inputs. The factor prices are insensitive to the factor-ratio, hence, no diminishing returns (as indicated by the Green Envelope).

If the Blue Line intersects with the Green Segment, the economy produces both inputs in the steady state, k_∞.

Furthermore,



Long-Run Rybczynski Theorem:

An increase in $sR/(n+\delta)$ increases k_∞, hence, the economy produces more capital-intensive inputs and less labor-intensive inputs.

From this, it is straightforward to show that:

Long-Run Heckscher-Ohlin Theorem:

Consider a two-country world, Home & Foreign, which differ only in the parameters, s (s^*), R (R^*), and n (n^*). If both countries are diversified in the steady state, Home exports the capital intensive inputs and Foreign exports the labor intensive inputs, if and only if $sR/(n+\delta) > s^*R^*/(n^*+\delta)$.

Notes:

- The assumption that both countries are diversified is implied if the differences between the two countries are sufficiently small.
- Findlay (1970) obtained this result for the case with two tradeable consumption goods and one nontradeable investment good, under the assumption that the factor intensity of the investment goods sector lies between the factor intensities of the two consumption goods sector.

- Stiglitz (1970) looked at the case with one consumption good and one investment good, both of which are tradeable and studied the likelihood of the economy staying in the diversification cone in steady state. The result depends, among other things, on the relative intensity of the two goods sectors.
- Matsuyama (1988) showed that, in the OLG, life-cycle saving framework, the aggregate saving depends on the population growth rate (as it changes the ratio of the young saver and the old retirees), and restated the Findlay result in the terms of the discount rate and the population growth rate.
- Ventura (1997) studied this setup with optimal savings by the representative agents, without assuming that the economy is diversified in steady state. The economy still passes through the diversification cone during transition. He used this model to address, among other things, the question of the East Asian Miracle; why these countries managed to grow so fast for so long. He argued that the export-led growth, accompanied by structural transformation from labor-intensive to capital-intensive industries might delay the standard capital deepening effect in these countries.
- In Part 3, we saw that the factor price insensitivity is not a robust result. But, the idea that the factor prices depend *less* on the local factor availability in a trading world is compelling. So, it would be interesting questions for future research to see how much of the above result can be intended to general factor proportion models with trade costs.

Growth and Trade: Innovation (unfinished)

Growth and Trade: Technology Diffusion (unfinished)

Short Run versus Long Run: Structural Adjustment Models (unfinished)

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